Statistical Physics & Condensed Matter Theory I: Exercise

Grassmann variables

With the adopted definitions for integration over Grassmann variables, many results look very much like those for bosonic variables. As an example, consider the delta function defined from an integral over an auxiliary (Grassmann) variable η:

$$\delta(\xi - \xi') \equiv \int d\eta\, e^{-\eta(\xi - \xi')}.$$

a)

Show that this yields the simple form $\delta(\xi - \xi') = -(\xi - \xi')$.

b)

Verify that this simple form has the correct behaviour, *i.e.* that for a general function f of one Grassmann variable, we have

$$\int d\xi'\, \delta(\xi - \xi') f(\xi') = f(\xi).$$