Statistical Physics & Condensed Matter Theory I: Exercise

The hesitant random walker

Mister Hesitant Onedim lives on a one-dimensional lattice made of infinitely many equidistant sites (the lattice distance being set to a). Being even more indecisive than the usual random walker, mister H. Onedim not only decides his walking direction randomly, but he also randomly decides *whether* he takes a step at all. Observing him for a while, you deduce that he follows these rules:

- **rule 1**: at each time interval δt, mister H. Onedim takes one step on the lattice with probability p ($0 \leq p \leq 1$), and remains in place with probability $1 - p$.
- **rule 2**: if he takes a step, the direction is uniformly distributed among all possible directions.

a) Write down the one-step time evolution equation obeyed by the probability $P_{r_1,t_1|r_0,t_0}$ of finding mister H. Onedim at site r_1 at time t_1, given that he was at site r_0 at time t_0.

b) Defining a scaling limit properly and taking the continuum limit, obtain the probability density (per unit length)

$$p(r_1, t_1|r_0, t_0) \equiv \lim a^{-1} P_{r_1,t_1|r_0,t_0}$$

of finding mister H. Onedim at a given position at a given time. Does he obey the diffusion equation? If yes, what is the value of the diffusion constant?

c) Consider now the specific case $p = 1/n$ where n is a positive integer. Find an explicit relationship between mister H. Onedim's probability density, and that of a (non-hesitating) random walker on an n-dimensional lattice.